Could This Be Random? Probably!

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Background, Purpose and Hypothesis:

People encounter events that deal with probability, statistics and randomness every day. An example of such an event is the shuffling of a deck of cards. A common type of shuffle is the riffle shuffle, in which the deck is split roughly in half and then weaved or 'riffled' back together.

The purpose of the experiment was to investigate how many riffle shuffles were needed to randomize a deck of 52 cards, and to investigate how effective different methods of measurement were for evaluating the 'randomness' of a set of discrete objects.

The hypotheses predicted:

A. "If consecutive shuffles bring the ordering of the deck closer to a random distribution, then there exists a number of shuffles where further shuffling will not bring the ordering any closer to a random distribution."
B. "Deck orderings and card occurrences, when measuring deck randomness, will not always agree."

Procedure:

1. A program that simulated a riffle shuffle through use of the Gilbert/Shannon/Reeds (GSR) Model was written.*
2. Another program that simulated a random re-ordering of a deck was written.*
3. The riffle shuffling program was tested for one to fifteen riffle shuffles
4. These results were written to data files and inserted into MS Excel spreadsheets where they were formatted and arranged into graphs.

The independent variable was the number of times a deck was shuffled; the control set was the result of the random shuffler; the dependent variable was the difference between the independent and control sets (the randomness), and the comparison between decks produced by different numbers of shuffles.

Observations and Conclusions:

After the experiment, it was seen that the amount of shuffles required to randomize a deck of cards depended greatly on the method of measurement. Hypothesis A was proven true; with both methods of measurement, there existed a number of shuffles after which, further shuffling did not reliably increase randomness. The exact number of shuffles to randomize a deck of 52 cards was shown by the experiment to be either three or seven, depending on how randomness was measured.

When randomness was measured using average card distribution, the decks were shown to become random in as few as three shuffles. This was due to the fact that average card distribution measures the probability that any card could show up in any location. It ignored isolated cases such as the probability of card 52 appearing in slot 1, which was very unlikely at three shuffles. At three shuffles, the majority of cards were equally likely to appear in any card slot, but this was not true for all cards.

Measuring randomness using rising sequence showed that the decks became random after seven shuffles. Since rising sequence measures the difference between the ordering of the deck before and after shuffling, the results were different from when randomness was measured using card distribution. This proved that hypothesis B was also correct.

The key difference between these two methods is that comparing distribution measures the average results of individual cards, whereas rising sequence measures the results of the entire deck ordering. Thus, the number of shuffles needed to sufficiently randomize a deck depends greatly on how 'random' is defined, and how it is measured.

When riffle shuffling a deck of cards, it takes only three shuffles until most of the cards are equally likely to appear in any slot. However, it takes seven shuffles to make every deck configuration equally likely. Therefore, when playing casually, three shuffles and a ‘cut’ of the deck (to prevent cheating by ‘floating’ cards near the top or bottom) are sufficient to randomize a deck of cards. When playing in tournaments on the other hand, seven shuffles are recommended to ensure that both a) every deck configuration is equally likely, and b) every card is equally likely to appear in any slot.

In conclusion, the experiment showed that the randomness of card distributions increased significantly until three shuffles, and the randomness of deck configurations increased significantly until seven shuffles.

<u>Other Applications:</u>

The results of this experiment would benefit card players throughout the world. Knowing the most efficient number of shuffles to perform on a deck of cards is very useful when playing many rounds of a game, such as poker. The programs created by the experimenter throughout this project could be used to show how well riffle shuffles and random re-orderings are modeled in computer programs.

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* Shuffling programs used in this project were written by, and are the sole property of Eddie Kim.