The Factorial Factor

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Background, Purpose and Hypothesis:

As a player of "Magic: The Gathering" Trading Card Game, I use card sleeves to protect my decks. However, I often wondered if the sleeves affected shuffling. In this experiment, my purpose was to investigate the effect of "Dragon Shield" card sleeves on riffle shuffling. My hypothesis is that the card sleeves will improve shuffling ability because the plastic reduces friction, and increases the gaps between individual cards.

The title, "The Factorial Factor," seems vague at first, but it is named with good reason. As previously mentioned, the project is about card shuffling. It can be said that a 'shuffle' is simply a random permutation of a deck of cards. How many permutations are there for a deck of n cards? The answer is $n!$ (n factorial).

Procedure:

Test variable: The absence/presence of card sleeves, the number of shuffles (only in Experiment B), ability of shuffler (only in Experiment A).

Affected variable: The randomness of the decks.

Controlled variables: The type of shuffles, the number of shuffles (only in

Experiment A), the type of cards, and the relative condition of the cards.

Experiment A:

1. Two test decks were created, one with sleeves and one without. The cards were labeled with the numbers 1-60 per deck and ordered as such.

2. A volunteer was then asked to perform ten riffle shuffles on each deck.

3. After each ten shuffles, each deck's 'randomness' was evaluated by using a method based on "rising sequence." This method consisted of counting adjacent pairs and then observing how many times it was necessary to search through the deck (front to back) while pulling out the cards in the original order.

4. Steps 2 and 3 were repeated for each of the six volunteers, and then the results were recorded and analyzed.

Experiment B:

1. Both the sleeved and non-sleeved decks were shuffled twice and then three times, then four, and so on, up to nine times. The decks were evaluated and reset after each shuffle (i.e. two shuffles and then evaluated and reset, three shuffles and then evaluated and reset).

2. The results were recorded and analyzed.

Observations/Conclusions:

"Experiment A" showed that, with a variety of shufflers, overall, sleeves will help bring the rising sequences closer to the optimum number, thirty, and the adjacent pairs closer to the optimum number, zero. "Experiment B" showed that sleeves made almost no difference when used by a veteran shuffler. The experiment also showed that, with decks of 60 cards, it takes only about four riffles to lower the adjacent pairs to the 1-3 range, and seven to optimize the rising sequences to the 27-33 range.

The experiments showed that sleeves are beneficial to bad shufflers, but not as useful for good shufflers. This was due to reasons such as card thickness and friction.

With an un-sleeved card, the edges are just as thick as the center of the card. However, a card sleeve is larger than the actual card. Thus, a sleeved card is thinner at the edge than the center. Therefore, when you have a deck of sleeved cards, there are large spaces between each card, allowing for easier grip. Plastic also gives less friction between cards, allowing the cards to slide together much more easily.

The key motion for a riffle shuffle is the moving of the thumbs up the edges of the cards. For a good shuffle, the cards should be interwoven close to one-over-one, not in clumps of cards. The card sleeves help a person achieve the 'riffle' motion far more successfully.

These factors make it easier for a newer player to shuffle using sleeves because they do not need a good grip to create an effective shuffle, whereas a veteran shuffler has a well-developed grip. Therefore, **card sleeves affect the quality of the shuffle with an amount inversely proportional to the shuffler's ability**.

Next Steps/Real World Applications:

The information from my experiment could benefit all players of "Magic: The Gathering" as well as people who play other Trading Card Games such as "Yu-Gi-Oh," "Duel Masters" or "Pokémon." The results from Experiment B could also help poker players, because the results display how many shuffles are needed to randomize a deck of cards.

If I were to continue this experiment, I would also like to measure and compare other relevant data such as hand size of shufflers, dominant hand of shuffler, and the edge that the cards were riffled on. I would also test the results of human shuffling compared to algorithm-based computer shuffling.

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